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Separable potentials and Coulomb interactions

D. HUSAIN
Department of Mathematics, Dacca University, Dacca-2, Pakistan
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Abstract. The scattering of two charged particles by a separable nonlocal potential and a repulsive Coulomb potential has been studied. Exact expressions have been derived for the scattering amplitude and an equivalent local potential.

Recently Cassola and Koshel (1968), Moiseiwitsch (1969) and Husain and Ali (1970) have studied the scattering of two uncharged particles by a separable, nonlocal potential. In this communication we will derive an exact expression for the scattering amplitude and an expression for an equivalent local potential for the scattering of two charged particles by a separable nonlocal potential.

The Schrödinger equation describing the motion of two charged particles via a nonlocal potential and a repulsive Coulomb potential is

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2}+k^2-\frac{l(l+1)}{r^2}-\frac{2nk}{r}\right)\mu_l(r) = \int_0^\infty K_l(r,r')\mu_l(r')\,\mathrm{d}r' \tag{1}$$

where $k^2 = 2mE/\hbar^2$, $n = 2mZe^2/\hbar^2k$, m is the reduced mass, Ze and e are the charges of the particles

If the potential $K_l(r, r')$ is separable

$$K_l(r,r') = \lambda q_l(r)\,q_l(r') \tag{2}$$

in that case the solution of equation (1) is

$$\mu_l(r) = F_l(r)+\lambda a_l\int_0^\infty \bar{G}_l(r,r')\,q_l(r')\,\mathrm{d}r' \tag{3}$$

where

$$a_l = \frac{\displaystyle\int_0^\infty F_l(r)\,q_l(r)\,\mathrm{d}r}{1-\lambda\displaystyle\int_0^\infty\int_0^\infty \bar{G}_l(r,r')\,q_l(r)\,q_l(r')\,\mathrm{d}r\,\mathrm{d}r'} \tag{4}$$

$$\begin{aligned}\bar{G}_l(r,r') &= \frac{1}{k}F_l(r)G_l(r') \qquad r'>r\\ &= \frac{1}{k}F_l(r')G_l(r) \qquad r'<r.\end{aligned} \tag{5}$$

$F_l(r)$ and $G_l(r)$ are the regular and irregular solutions of the equation (Goldberger and Watson 1964)

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2}+k^2-\frac{l(l+1)}{r^2}-\frac{2nk}{r}\right)g_l(r) = 0 \tag{6}$$

such that, as $r \to \infty$,

$$\begin{aligned} F_l(r) &\to \sin(kr - \tfrac{1}{2}l\pi - n\ln 2kr + \sigma_l) \\ G_l(r) &\to \cos(kr - \tfrac{1}{2}l\pi - n\ln 2kr + \sigma_l) \\ \sigma_l &= \arg\Gamma(l+1+in). \end{aligned} \tag{7}$$

From the asymptotic form of equation (3) it is found that the Coulomb-corrected 'nuclear' phase shift is given by

$$\tan\delta_l = -\frac{\lambda}{k}\frac{\left(\int_0^\infty F_l(r)q_l(r)\,\mathrm{d}r\right)^2}{1-\lambda\int_0^\infty\int_0^\infty \bar{G}_l(r,r')q_l(r)q_l(r')\,\mathrm{d}r\,\mathrm{d}r'}. \tag{8}$$

The above expression is similar to that derived by Harrington (1965).

We will now derive an expression from the equivalent local potential for scattering of two charged particles by a separable, nonlocal potential.

Fiedeldey (1967) has shown that an equivalent local potential $U_l(r)$ may be written as

$$U_l(r) = -\frac{f_l''(r)}{f_l(r)} + 2\left(\frac{f_l'(r)}{f_l(r)}\right)^2 + \frac{1}{f_l^2(r)}\int_0^\infty K_l(r,r')\{\mu_l'(r)\nu_l(r') - \mu_l(r')\nu_l'(r)\}\,\mathrm{d}r' \tag{9}$$

where

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}r}\{f_l(r)\}^2 &= \mu_l''(r)\nu_l(r) - \mu_l(r)\nu_l''(r) \\ f_l(r) &\to 1 \quad \text{as} \quad r\to\infty. \end{aligned} \tag{10}$$

$\mu_l(r)$ and $\nu_l(r)$ are the regular and irregular solutions of equation (1) such that, as $r \to \infty$,

$$\mu_l'(r)\nu_l(r) - \mu_l(r)\nu_l'(r) \to 1. \tag{11}$$

In the case of a separable, nonlocal potential (equation (2)) we find that

$$\nu_l(r) = \frac{\lambda a_l c_l}{k^2+\lambda^2 a_l^2 c_l^2}\mu_l(r) - \frac{1}{k+\lambda b_l c_l}\omega_l(r) \tag{12}$$

$$\omega_l(r) = G_l(r) + \lambda b_l\int_0^\infty \bar{G}_l(r,r')\,q_l(r')\,\mathrm{d}r' \tag{13}$$

$$b_l = \frac{\int_0^\infty G_l(r)\,q_l(r)\,\mathrm{d}r}{1-\lambda\int_0^\infty\int_0^\infty \bar{G}_l(r,r')\,q_l(r)\,q_l(r')\,\mathrm{d}r\,\mathrm{d}r'} \tag{14}$$

$$c_l = \int_0^\infty F_l(r)q_l(r)\,\mathrm{d}r \tag{15}$$

$$\{f_l(r)\}^2 = \frac{1}{k+\lambda b_l c_l}\left(k + \lambda b_l\int_0^r F_l(r')q_l(r')\,\mathrm{d}r' + \lambda a_l\int_r^\infty G_l(r')q_l(r')\,\mathrm{d}r'\right) \tag{16}$$

and

$$U_l(r) = -\frac{q_l'(r)}{q_l(r)}\frac{f_l'(r)}{f_l(r)} + 3\left(\frac{f_l'(r)}{f_l(r)}\right)^2 + \frac{3\lambda\{a_l G_l'(r) - b_l F_l'(r)\}q_l(r)}{2(k+\lambda b_l c_l)\{f_l(r)\}^2}. \tag{17}$$

From equations (4), (14) and (16) it is clear that, as $r \to 0, f_l(r) \to 1$, that is, the actual wavefunction and the wavefunction corresponding to the local potential coincide at the origin.

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